

Mathematical Science I Ken-Ichi Nakamura (Room 386)
Homework 1 (Due: April 27)

Problem 1 Prove the following by using ε - N argument:

Let $\{a_n\}_n$, $\{b_n\}_n$ and $\{c_n\}_n$ be real sequences. Suppose that for every $n \in \mathbb{N}$, we have

$$b_n \leq a_n \leq c_n$$

and also suppose that

$$\lim_{n \rightarrow \infty} b_n = l = \lim_{n \rightarrow \infty} c_n$$

for some $l \in \mathbb{R}$. Then

$$\lim_{n \rightarrow \infty} a_n = l.$$

Problem 2 Prove the following:

- (1) If a sequence of real numbers converges, then it is bounded.
- (2) If a sequence of real numbers converges, then it is a Cauchy sequence.