

Mathematical Science I Ken-Ichi Nakamura (Room 386)
Homework 2 (Due: May 11)

Problem 1 Let $f, g : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ be given by

$$f(x) = x \sin \frac{1}{x}, \quad g(x) = \sin \frac{1}{x}.$$

Prove that $\lim_{x \rightarrow 0} f(x) = 0$, while $\lim_{x \rightarrow 0} g(x)$ does not exist.

Problem 2 Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be such that f is continuous at $a \in \mathbb{R}$, and g is continuous at $f(a)$. Prove that the composite function $g \circ f$ is continuous at a .

Problem 3 Prove that a monotone increasing (not necessarily continuous) function $f : I \rightarrow \mathbb{R}$ (that is, $f(x_1) \leq f(x_2)$ for $x_1 \leq x_2$) is integrable on I .