Application of the nonlinear Galerkin FEM method to the solution of the reaction diffusion equations

Jan Mach\textsuperscript{1}

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1 Abstract

Complex long-term dynamics of reaction-diffusion problems requires finer approach leading to quantitatively reliable numerical schemes, for which the error estimate constant does not grow exponentially in time. One of approaches is known as the nonlinear Galerkin method and was introduced by M. Marion and R. Temam [3].

The idea of the method is further extended to finite element method in [4]. The motivation for this method was to capture the effect of some of the terms that are neglected in the usual Galerkin method. In our talk we present an application of this method to the numerical solution of the selected reaction-diffusion equations. Our test systems include the Gray-Scott model [2, 1], the Brusselator model [5] and the phase-field model.

We restrict ourselves to one spatial dimension, $\Omega \subset \mathbb{R}$, and consider two levels of discretization, one with the mesh parameter $2h$ and the second twice finer with mesh parameter $h$. Generalization to higher number of discretization levels is possible. Number of mesh nodes $n$ is required to be odd, thus $n = 2m + 1$ for $m \in \mathbb{N}$.

The computational results demonstrate properties of the method. We performed comparison with the finite-difference method and present overview of our computational results for selected reaction-diffusion equations. We performed quantitative comparison of numerical solutions, comparison of CPU-time consumption and measurement of the experimental order of convergence.

\textsuperscript{1}Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague.
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References


